The concept of field in the history of electromagnetism

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ET2011-XXVII Riunione Annuale dei Ricercatori di Elettrotecnica

Bologna 16-17 giugno 2011

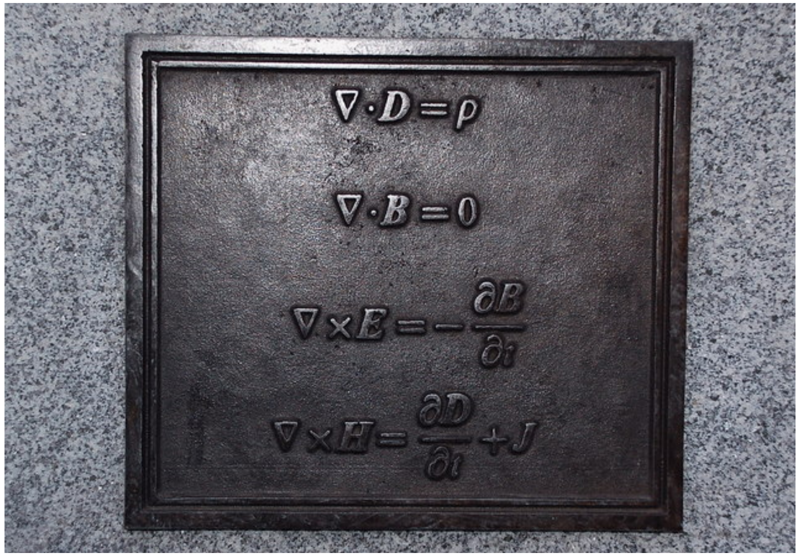
Celebration of the 150th Birthday of Maxwell's Equations

150 years ago (on March 1861) a young **Maxwell** (30 years old) published the first part of the paper

On physical lines of force

in which he wrote down the equations that, by bringing together the physics of electricity and magnetism, laid the foundations for electromagnetism and modern physics.

Statue of Maxwell with its dog Toby.
Edinburgh, George Street.



Plaque on E-side of the statue.

Talk Outline

- **A brief survey of the birth of the electromagnetism: a long and intriguing story**
- **A rapid comparison of Weber's electrodynamics and Maxwell's theory: "direct action at distance" and "field theory"**

General References

E. T. Wittaker, *Theories of Aether and Electricity*, Longam, Green and Co., London, 1910.

O. Darrigol, *Electrodynamics from Ampère to Einste in*, Oxford University Press, 2000.

O. M. Bucci, *The Genesis of Maxwell's Equations*, in "History of Wireless", T. K. Sarkar et al. Eds., Wiley-Interscience, 2006.

Magnetism and Electricity

In 1600 **Gilbert** published the "*De Magnete, Magneticisque Corporibus, et de Magno Magnete Tellure*" (On the Magnet and Magnetic Bodies, and on That Great Magnet the Earth).

- The Earth is magnetic (Μαγνησία, Magnesia ad Sipylum) and this is why a compass points north.

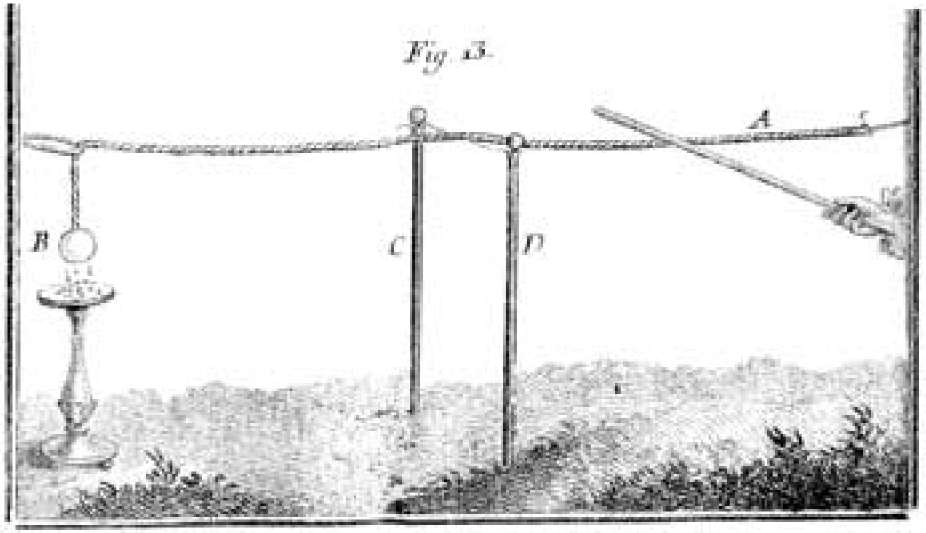
- In a quite large class of bodies (glass, sulphur, …) the friction induces the same effect observed in the amber (Ηλεχτρον, Elektron). Gilbert gave to it the name "*electricus".*

- Gilbert remarked many distinction between the magnetic and the electric forces.
  For instance, the lodestone requires no stimulus of friction such as it is needed to stir glass and sulphur into electrical activity.

William Gilbert
(England, 1540-1603)

Electric Current

In 1729 **Gray** discovered the “passage of electricity” through metals.



Stephen Gray
(England, 1666-1736)

Vitreous and Resinous Electricity

In 1733 **Du Fay** discover the two kinds of electricity: *vitreous* and *resinous* (for instance, sulfur is vitreous and amber is resinous).

We say nowadays *positive* and *negative electricity,* terms coined independently by **Watson** (1746) and **Franklin**.

Charles François
de Cisternay Du Fay
(France, 1698–1739)

William Watson
(England, 1715 –1787)

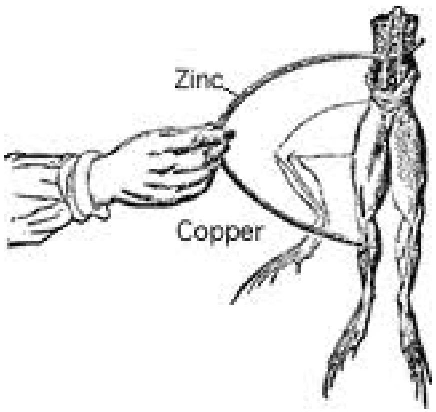
Benjamin Franklin
(United Staes, 1706-1790)

Independently, **Franklin** (1747) and **Watson** (1746) stated the principle of conservation of the quantity of electrical charge.

Electric Currents Chemically Generated

In 1780 **Galvani** discovered the possibility to generate chemically electric currents.

The metals used to mount and dissect the leg of frogs induced electric currents in their nerves.

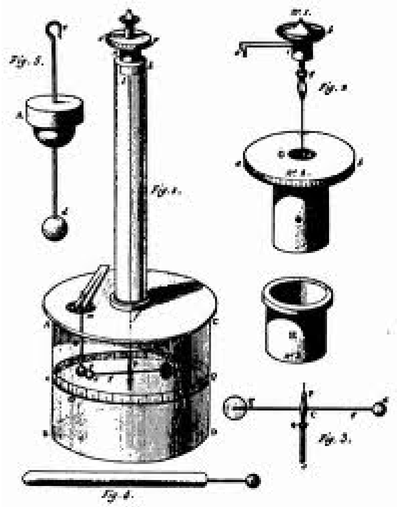


Luigi Galvani
(Italy, 1737-1798)

He gave rise to an interest in electricity created by the contacts between different materials.

Coulomb's Laws

In 1785 **Coulomb** formulated the law governing the force acting between electric charges and the law governing the force acting between magnetic poles.



Charles Augustin de Coulomb (France, 1736-1806)

- The electricity is composed of two electric fluids that can be separated: the parts of the same fluid repelling each other according to the inverse square of the distance, and attracting the parts of the other fluid according to the same inversion square law.

- The magnetism is also composed of two fluids, the *austral* and *boreal,* permanently imprisoned within the molecules of magnetic bodies, which obey the same inverse square law.

Direct Action at Distance *versus* Aether

"Until the seventeenth century the only influence which was known to be capable of passing from star to star was that of light. **Newton** added to this the **force of gravity**; …" (Wittaker)

Newton attempted to account for gravitation by differences of pressure in an *aether*, but he did not publish his theory because he was not able from experiments and observations to give a satisfactory account of this medium.

The only *aether* that survived is that invented by **Huygens** (1690): the medium that penetrates all matter and is present even in all so-called vacuum, in which the propagation of light takes place.

Isaac Newton
(England, 1643 - 1727)

Christian Huygens
(Holland, 1629-1695)

After **Coulomb** it was recognized that the power of communicating directly across vacuous regions was possessed also by the **electric** and **magnetic interactions**.

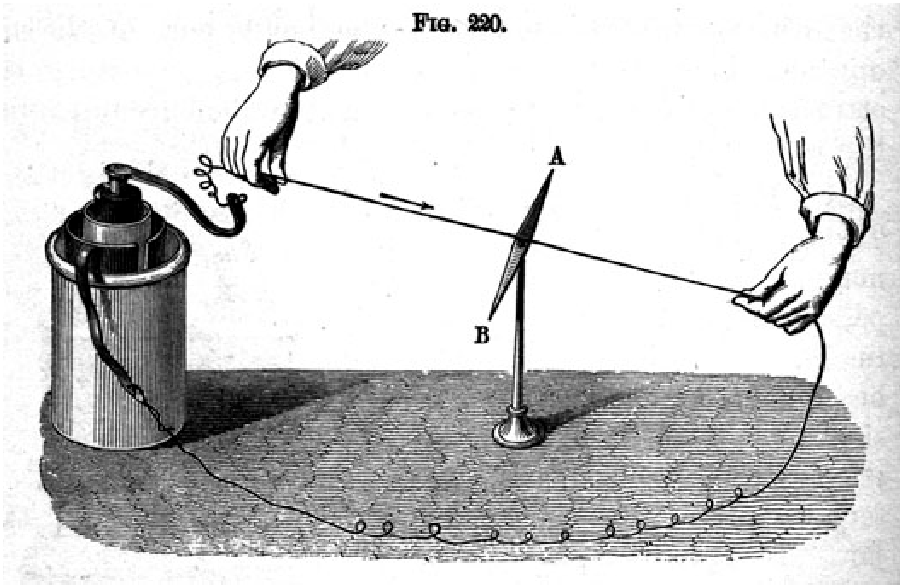
Volta's Pila

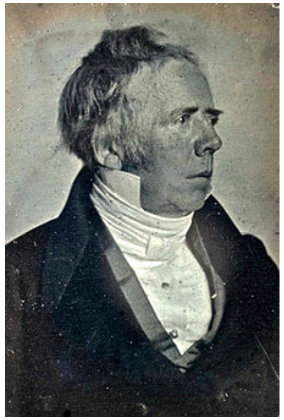
In 1792 **Volta** invented the "pila".

Alessandro Volta
(Italy, 1745-1827)

Oersted's Experiment

On 1820 **Oersted** discovered that a *magnetic needle* is acted on by a *voltaic current*.

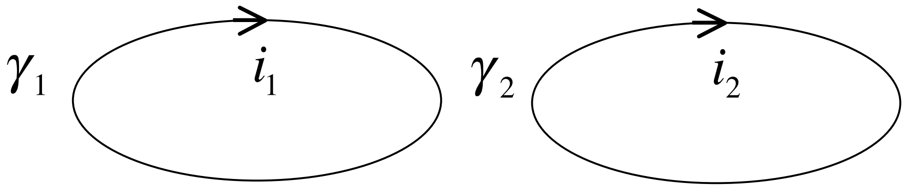




Hans Oersted
(Denmark, 1777-1851)

Ampère's Law

On 1820 **Ampère** demonstrated that parallel wires carrying currents attract or repel each other. This laid the foundation of **electrodynamics**.



André-Marie Ampère
(France, 1775-1836)

$$\mathbf{F}_{21} = -\frac{\mu_0}{4\pi} i_1 i_2 \oint_{\gamma_1} \oint_{\gamma_2} \frac{\hat{\mathbf{r}}_{12}}{r_{12}^2} \left(d\mathbf{s}_1 \cdot d\mathbf{s}_2 \right)$$

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

(in nowadays notation and International System (*SI*) Units

Ohm's Law

In 1827 **Ohm** published the book "*Die galvanische Kette, mathematisch bearbeitet*" (The Galvanic Circuit Investigated Mathematically) where his law is formulated.

$$S = \gamma E$$

current flowing in a wire → S

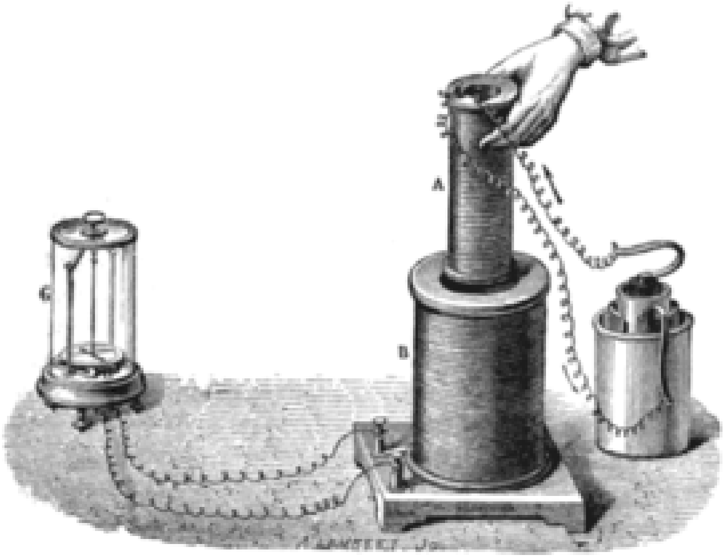
difference of "electroscopic force" at wire terminals → E

Ohm's law connected the theory of currents with the older theory of electrostatic.

George Simon Ohm
(Germany, 1789-1854)

Electromagnetic Induction

In 1831 **Faraday** discovered the electromagnetic induction.



Michael Faraday
(England, 1791-1867)

“Distinct conversion of Magnetism into Electricity” (Faraday)

Electrotonic state

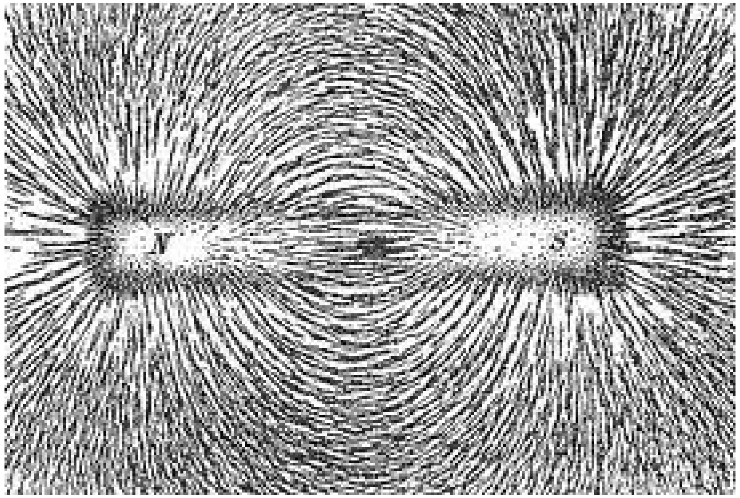
“This state does not manifest itself by any known phenomena as long as it is undisturbed, but any change in this state is indicated by a current or tendency towards a current.” (J. C. Maxwell)

Faraday's Law

Faraday determined the law of induction of currents by introducing a new way of representing the magnetic field.

Lines of magnetic force

"By magnetic curves, I mean the lines of magnetic forces, however modified by the juxtaposition of poles, which could be depicted by iron fillings; or those to which a very small magnetic needle would form a tangent." (Faraday, Exp. Res. § 114)



With these lines Faraday conceived all space to be filled.

"whether the wire moves directly or obliquely across the lines of force, in one direction or another, it sums up the amount of the forces represented by the lines it has crossed", so that "the quantity of electricity thrown into a current is directly as the number of curves intersected". (Faraday, Exp. Res. § 114)

The induced electromotive force is proportional to the number of the unit lines of magnetic force intersected by the wire per second.

Lenz's Law

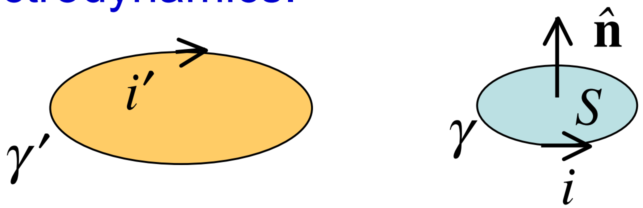
In 1834 **Lenz** formulated his law which can be enunciated as it follows:

"when a conducting circuit is moved in a magnetic field, the induced current flows in such a direction that the ponderomotive forces on it tend to oppose the motion".

Heinrich Friedrich Emil Lenz
(Baltic German, 1804-1865)

Neumann's Law of Induction

In 1845 **Neumann** deduced the law of induction of currents from Lenz's law by using the Ampère's electrodynamics.



The electromotive force induced in γ by i' is proportional to the time-rate of variation of the potential of the ponderomotive force acting on γ due to i' and when it is traversed by a unit current, i = 1, (*SI* Units)

$$U = i\int_S \mathbf{B}'\cdot\hat{\mathbf{n}}dS = \frac{\mu_0}{4\pi}ii'\oint_\gamma\oint_{\gamma'}\frac{d\mathbf{s}\cdot d\mathbf{s}'}{|\mathbf{r}-\mathbf{r}'|}$$

Franz Neumann
(Germany, 1798-1895)

Neumann introduced the **vector potential** (SI Units)

$$\mathbf{A}'(\mathbf{r}) = \frac{\mu_0}{4\pi}\oint_{\gamma'}\frac{i'd\mathbf{s}'}{|\mathbf{r}-\mathbf{r}'|}$$

The electromotive force induced along any-circuit element ***ds*** by any alteration in the current which give arise to **A**' is

$$\frac{\partial\mathbf{A}'(\mathbf{r};t)}{\partial t}\cdot d\mathbf{s}$$

Gauss's Intuitions

In 1835 **Gauss** suggested that electromagnetic theory should be derived from a single equation for the force between moving charges, which should be function of the relative velocity of the charges besides the distance between them.

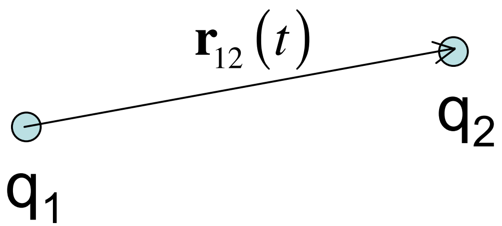
Karl Friederich Gauss
(Germany, 1777-1855)

In a letter to Weber (on March 19, 1845) Gauss remarked that this force should also include retardation:

the "electric action" should propagate between the charges with a finite velocity.

Weber's Electrodynamics: Direct Action at Distance

In 1846, **Weber** proposed a theory that unified Coulomb's law, Ampère's electrodynamics and electromagnetic induction.

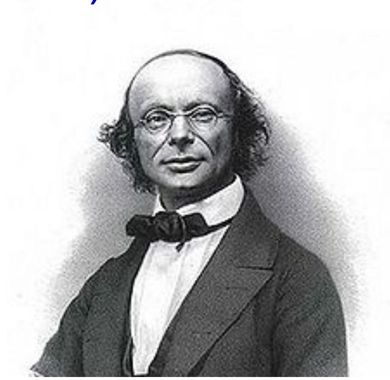


$$\mathbf{f}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}_{12}\left[1 - \frac{1}{2a^2}\left(\frac{dr_{12}}{dt}\right)^2 + \frac{1}{a^2} r_{12}\frac{d^2 r_{12}}{dt^2}\right]$$

Force exerted, in vacuum, between the point charges q_1 and q_2
(in the modern vector notations and SI units)

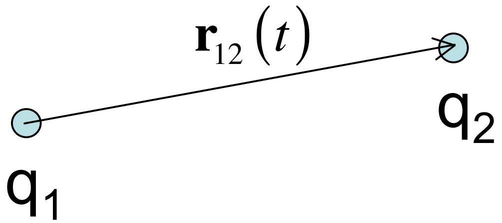
$$\mathbf{f}_{12} = -\mathbf{f}_{21}$$

The constant "*a*" is the ratio of the electromagnetic and electrostatic units of the electric charge in the CGS system.



Wilhelm Eduard Weber
(Germany, 1804-1891)

Weber's Electrodynamics: Direct Action at Distance



$$\mathbf{f}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}_{12}\left[1 - \frac{1}{2c^2}\left(\frac{dr_{12}}{dt}\right)^2 + \frac{r_{12}}{c^2}\frac{d^2 r_{12}}{dt^2}\right]$$

In 1855 Weber in collaboration with Kohlrausch measured the constant "*a*"

$$a = 3.1\times10^8\ m/s \approx c$$

This value is essentially the same as the known value of the velocity of the light in air at that time, measured in 1849 by **Fizeau**.

Hippolyte Fizeau
(France, 1819-1896)

Friedrich Kohlrausch
(Germany, 1809-1858)

Weber has been the first to propose a comprehensive explanation of both electrodynamics and electromagnetic induction in the classic framework of a "**direct action at a distance**" between charges and currents.

Retarded Scalar Potential

In a fragmentary note, written in 1853, **Riemann**, a Gauss's pupil, proposed to replace the Poisson's equation for the electrostatic potential (in SI units)

$$\nabla^2 V + \frac{1}{\varepsilon}\rho = 0$$

by the equation

$$\nabla^2 V - \frac{1}{c^2}\frac{\partial^2 V}{\partial t^2} + \frac{1}{\varepsilon}\rho = 0$$

This is in agreement with the view now is accepted as correct, but it was too slight to serve as the basis for a complete theory of the electromagnetism.

Siméon-Denis Poisson
(France, 1781-1840)

Georg Friedrich Bernhard Riemann
(Germany, 1826-1866)

"On the Motion of Electricity in Wires"

In 1855 **Kelvin** investigated the transients in long cables assuming that the magnetic effects were negligible. The work of Kelvin was followed by a celebrated paper of Kirchhoff.

In 1857 **Kirchhoff** published "*On the Motion of Electricity in Wires*".

THE

LONDON, EDINBURGH AND DUBLIN

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

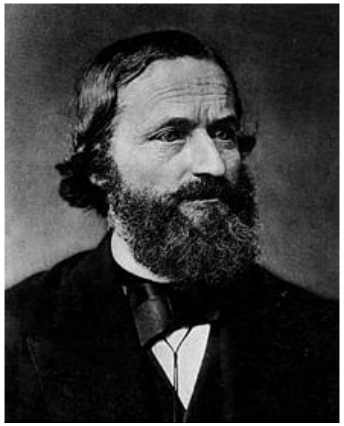
[FOURTH SERIES.]

JUNE 1857.

LIV. *On the Motion of Electricity in Wires.*
By G. KIRCHHOFF*.

I HAVE attempted to establish a general theory of the motion of electricity in an infinitely thin wire, by assuming certain facts which are observed in constant currents, and in currents whose intensity alters but slowly, to be universally valid. I will here develope this theory, and show its application to some cases of a simple nature.

William Thomson
(Lord Kelvin)
(England, 1824-1907)



Gustav Robert
Kirchhoff
(Germany, 1824-1887)

"On the Motion of Electricity in Wires"

By using Weber's electrodynamics and the charge conservation law Kirchhoff foresaw before Maxwell that an electric wave propagates along a conducting wire with the velocity of light in vacuum.

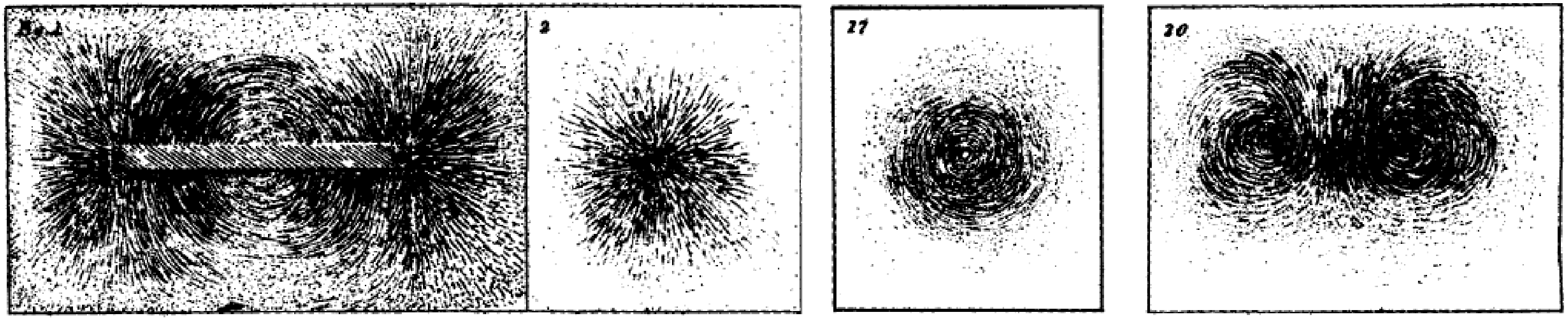
. . .

The velocity of propagation of an electric wave is here found to be $= c$ hence it is independent of the cross section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41950 German miles in a second, hence very nearly equal to the velocity of light *in vacuo*.

. . .

Faraday's Point of View: the Lines of Force

In 1852 **Faraday** published *"On the Physical Character of the Lines of Magnetic Force".*



The subtler kind of matter called *ether* did not exist. Masses, charged bodies, magnets and currents emanate "lines of forces" into the "empty" space surrounding them, through which they interact.

Faraday had no mathematical or mechanical preconceptions and his theory mostly reflected patient experimental explorations.

Faraday's Point of View: the Lines of Force

Faraday's notion of "lines of force" transcended the dichotomy between *direct action at distance* and *action through a medium*.

Faraday's researches were not motivated by the elimination of direct action at distance:

- He regarded an interaction via lines of force as direct action at a distance, whenever no matter contributed to the transmission of the force.
- He expected the interaction to take time, but not because a subtle medium or ether was involved.
- The reason of the retardation was the physical nature of the lines of force.
- The light was a transverse vibration of the lines of force.

Analogies

Kelvin was originally a mathematician with a strong background in analytical mechanics. His practical bent did not result from familiarity with the laboratory, but from Scottish Common Sense philosophy and interactions with engineers.

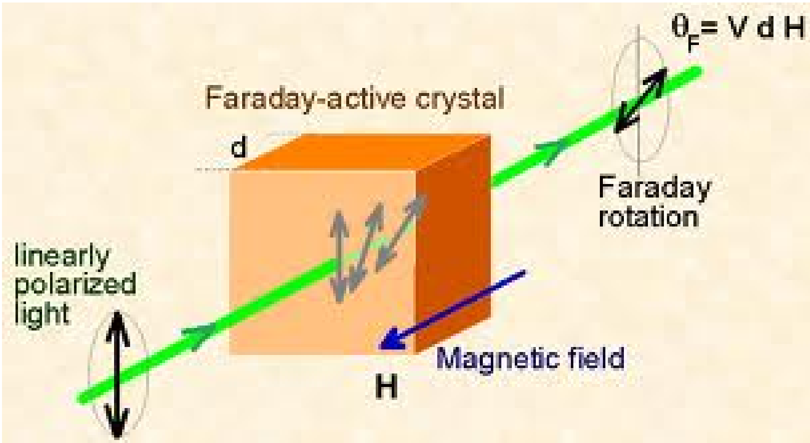
In **1841 Kelvin** studied the analogies between the distribution of electrostatic force, in a region containing electrified conductors, and the distribution of the flow of heat.

In **1846 Kelvin** investigated the analogies between electric and magnetic forces and strain in elastic solids.

By using analogies Kelvin translated direct action at distance explanations into explanations in terms of Faraday's conducting power for the line of force.

Faraday's Rotation

In 1845 **Faraday** also discovered the rotation of the plane of polarization of a linearly polarized light beam.
The polarization changes when the light propagates along the direction of an applied magnetic field in magneto-optical materials.



Light interacts with magnetic fields !

In **1856 Kelvin** constructed a dynamical theory of magnetism to explain Faraday's rotation. Magnetic force is <u>not</u> an interaction of *<u>static magnetic poles</u>* or *<u>stationary microscopic electric currents</u>* but a force between microscopic dynamic rotations: molecular vortices (Amperian point of view !).

Kelvin's Point of View: Fluid of Molecular Vortices

"In a notebook entry of 1858 Thomson did speculate on a general picture of *aether* and matter. He imagined a universal fluid with myriads of rotating … eddies … ." (Darrigol)

Electricity corresponds to the less disturbed parts of the fluid between the vortices.

Electric current alters the rotation of the vortices as a string pulled between two adhering wheels.

Magnetic attractions result from the centrifugal force of the vortices combined with the pressure of the fluid.

Magnetism is the alignment of the axes of the vortices.

Electromagnetic induction corresponds to the storage of momentum in the oriented vortices.

The **gyrostatic rigidity** of the vortices permits transverse vibrations of the medium to be identified with light.

Faraday's effect results from the influence of vortex motions on transverse vibrations of the medium.

Kelvin ended with a prophetical remark: "A complete dynamical illustration of magnetism and electromagnetism seems not at all difficult or far off."

Electromagnetic Field Theory

"Faraday and Thomson invented electromagnetic field theory: they introduced new theoretical entities in the space between electric and magnetic sources and elaborated powerful techniques for investigating the properties of these entities." (Darrigol)

Maxwell's Equations

- In 1855 and 1856 **Maxwell** published "*On Faraday's lines of force*"

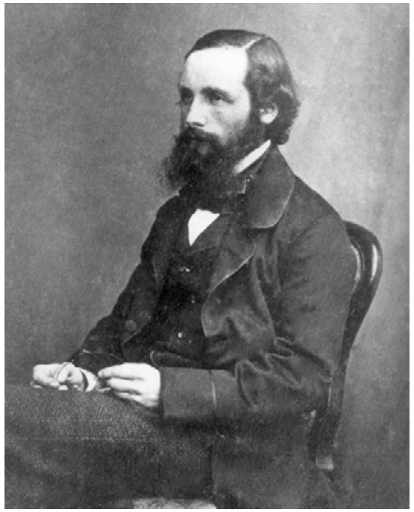
  Transactions of the Cambridge Philosophical Society, Vol. X, Part I 1855; Part II 1856.

- In 1861 and 1862 **Maxwell** published "*On Physical lines of force*"

  Phil. Mag., XXI, pp. 161-175, 281-291, 338-348, 1861 (Parts I and II); Phil. Mag., XXIII, pp. 12-25, 85-95, 1862 (Parts III and IV).

- In 1865 **Maxwell** published "*A Dynamical Theory of the Electromagnetic Field*"

  Phil. Trans. Roy. Soc., vol. 155, 1865, pp. 459-512.



James Clerk Maxwell
(Scotland, 1831-1879)

[From the *Transactions of the Cambridge Philosophical Society*, Vol. X. Part I.]

VIII. *On Faraday's Lines of Force.*

[Read *Dec.* 10, 1855, and *Feb.* 11, 1856.]

At very beginning of the paper he states that the purpose of the work is:

"… to show how, by a strict application of the ideas and methods of Faraday, the connection of the very different orders of phenomena which he has discovered can be clearly placed before the mathematical mind."

In the first part Maxwell stated the laws for electrostatic, magnetostatic and stationary currents by following Faraday, through the analogy with the hydrodynamics of a fluid in steady motion through a resisting medium.

The electricity, magnetism and currents are not real fluids, so the "fluid" is only a vehicle for the idea.

"On Faraday's Lines of Force"

In the second part by a full exploitation of the results of vector analysis, he stated the laws of electromagnetism and electromagnetic induction for **closed currents** (in nowadays vector notation):

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \textit{(I)}$$

$$\mathbf{H} = \vec{\mu}^{-1}\mathbf{B} \quad \textit{(II)}$$

$$\mathbf{j} = \nabla \times \mathbf{H} \quad \textit{(III)}$$

$$\mathbf{j} = \vec{\sigma}\mathbf{E},\ \mathbf{E} = -\nabla V + \mathbf{E}_i \quad \textit{(IV)}$$

V is the electrostatic potential

$$\mathbf{E}_i = -\partial \mathbf{A} / \partial t \quad \textit{(VI)}$$

Maxwell also provides the expression for the potential energy of a closed current in a magnetic field from which all the dynamical actions can be derived

$$U = i\oint_\gamma \mathbf{A}' \cdot d\mathbf{l} \quad \textit{(V)}$$

Electrotonic State

The vector potential is a measure of Faraday's *electrotonic state:* $\mathbf{A}'(\mathbf{r};t)\cdot d\mathbf{s}$ represents the total number of unit line of magnetic force which have passed across the line element $d\mathbf{s}$ prior to the instant t.

“On Faraday’s Lines of Force”

At the end of the paper Maxwell wrote:

…

In these six laws I have endeavoured to express the idea which I believe to be the mathematical foundation of the modes of thought indicated in the *Experimental Researches*. I do not think that it contains even the shadow of a true physical theory; in fact, its chief merit as a temporary instrument of research is that it does not, even in appearance, *account for* anything.

There exists however a professedly physical theory of electro-dynamics, which is so elegant, so mathematical, and so entirely different from anything in this paper, that I must state its axioms, at the risk of repeating what ought to be well known. It is contained in M. W. Weber's *Electro-dynamic Measurements*, and may be found in the Transactions of the Leibnitz Society, and of the Royal Society of Sciences of Saxony*. …

“On Faraday’s Lines of Force”

…

From these axioms are deducible Ampère’s laws of the attraction of conductors, and those of Neumann and others, for the induction of currents. Here then is a really physical theory, satisfying the required conditions better perhaps than any yet invented, and put forth by a philosopher whose experimental researches form an ample foundation for his mathematical investigations. What is the use then of imagining an electro-tonic state of which we have no distinctly physical conception, instead of a formula of attraction which we can readily understand? I would answer, that it is a good thing to have two ways of looking at a subject, and to admit that there *are* two ways of looking at it. Besides, I do not think that we have any right at present to understand the action of electricity, and I hold that the chief merit of a temporary theory is, that it shall guide experiment, without impeding the progress of the true theory when it appears. There are also objections to

…

Maxwell decisively claims the opportunity of looking for another visions of the physical word.

"On Physical Lines of Force" Part I.

In May 1857, after reading Kelvin's 'new lights' on the Faraday's effect and molecular vortices, Maxwell wrote to his friend *Cecil Monro*:

"*This was a wet day & I have been grinding at many things and lately during this letter at a Vortical theory of magnetism and electricity which is very crude but has some merits, so I spin & spin.*"

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[FOURTH SERIES.]

MARCH 1861.

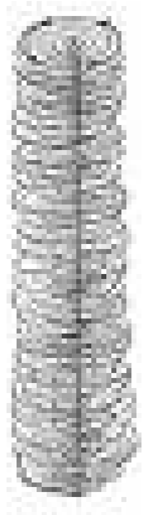
XXV. *On Physical Lines of Force.* *By* J. C. MAXWELL, *Professor of Natural Philosophy in King's College, London**.

PART I.—*The Theory of Molecular Vortices applied to Magnetic Phenomena.*

IN all phenomena involving attractions or repulsions, or any forces depending on the relative position of bodies, we have to determine the *magnitude* and *direction* of the force which would act on a given body, if placed in a given position.

“On Physical Lines of Force” Part I.

Maxwell extended Kelvin’s idea of representing Faraday’s magnetic field lines in terms of molecular vortices. To each line corresponds a rigid vortex and the entire distribution of magnetic field is represented by a vortical fluid.



The magnetic intensity force **H** corresponds to the angular velocity of the fluid, μ to the mass density, whence the field induction **B = μH** is related to the angular momentum; *p* is the isotropic pressure of the fluid. The energy density of the rotating fluid is written as $\mathbf{H}\cdot\mathbf{B}/2$.

By applying to this fluid the laws of continuum mechanics, Maxwell obtained the following expression for the force density in a homogeneous medium with permeability μ (in nowadays vector notation):

$$\mathbf{f} = \mathbf{H}\nabla\cdot(\mu\mathbf{H}) + \frac{\mu}{2}\nabla\left(\mathbf{H}^2\right) + (\nabla\times\mathbf{H})\times(\mu\mathbf{H}) - \nabla p$$

XLIV. *On Physical Lines of Force.* *By* J. C. Maxwell, *Professor of Natural Philosophy in King's College, London*†.

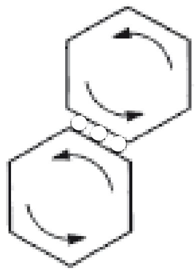
[With a Plate.]

Part II.—*The Theory of Molecular Vortices applied to Electric Currents.*

...

I have found great difficulty in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction about parallel axes. The contiguous portions of consecutive vortices must be moving in opposite directions; and it is difficult to understand how the motion of one part of the medium can coexist with, and even produce, an opposite motion of a part in contact with it.

...



How is it possible to go beyond the incongruence due to opposite rotation directions of adjacent vortices?

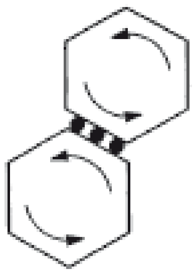
Idle Wheel

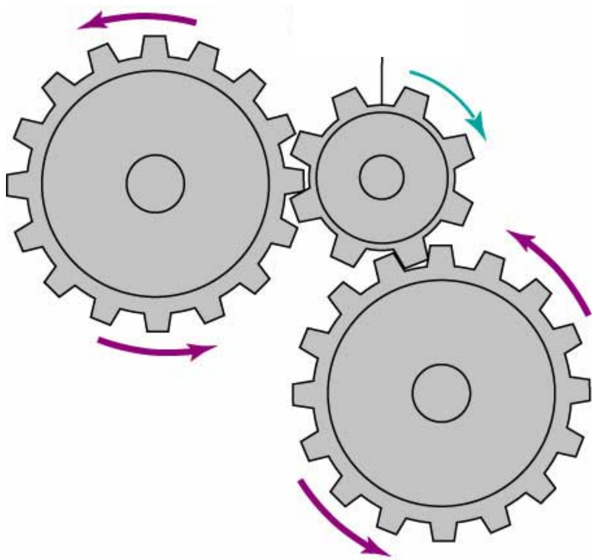
...

In mechanism, when two wheels are intended to revolve in the same direction, a wheel is placed between them so as to be in gear with both, and this wheel is called an "idle wheel." The hypothesis about the vortices which I have to suggest is that a layer of particles, acting as idle wheels, is interposed between each vortex and the next, so that each vortex has a tendency to make the neighbouring vortices revolve in the same direction with itself.

...

A layer of particles (for instance, spheres), acting as "idle wheel", is interposed between contiguous vortices to convey motion from one to the other vortex without changing their speed or direction of motion.





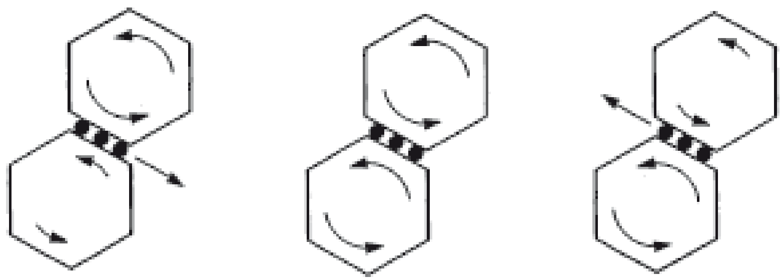
Academy Artworks

"On Physical Lines of Force" Part II.

…

In mechanism, the idle wheel is generally made to rotate about a *fixed* axle; but in epicyclic trains and other contrivances, as, for instance, in Siemens's governor for steam-engines*, we find idle wheels whose centres are capable of motion. In all these cases the motion of the centre is the half sum of the motions of the circumferences of the wheels between which it is placed. Let us examine the relations which must subsist between the motions of our vortices and those of the layer of particles interposed as idle wheels between them.

…



Whenever two contiguous vortices do not rotate at the same speed, the particles between them must shift laterally.

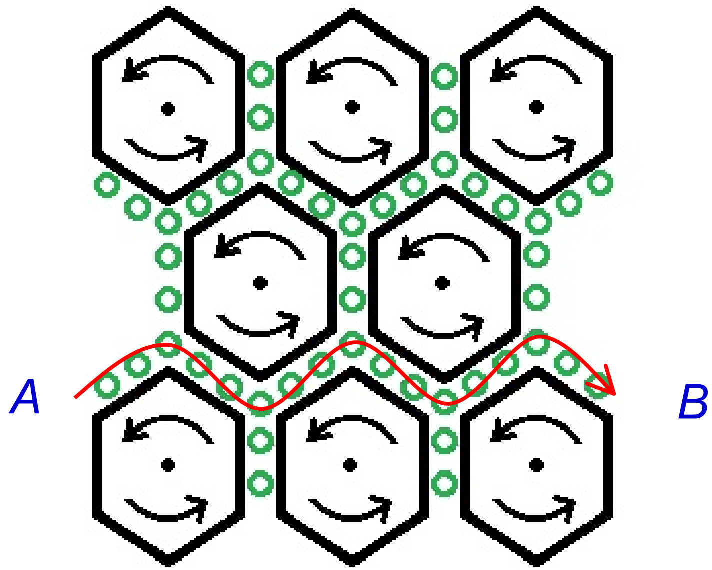
"On Physical Lines of Force" Part II.

. . .

We shall in the first place examine the process by which the lines of force are produced by an electric current.

Let A B, Pl. V. fig. 2, represent a current of electricity in the direction from A to B. Let the large spaces above and below A B represent the vortices, and let the small circles separating the vortices represent the layers of particles placed between them, which in our hypothesis represent electricity.

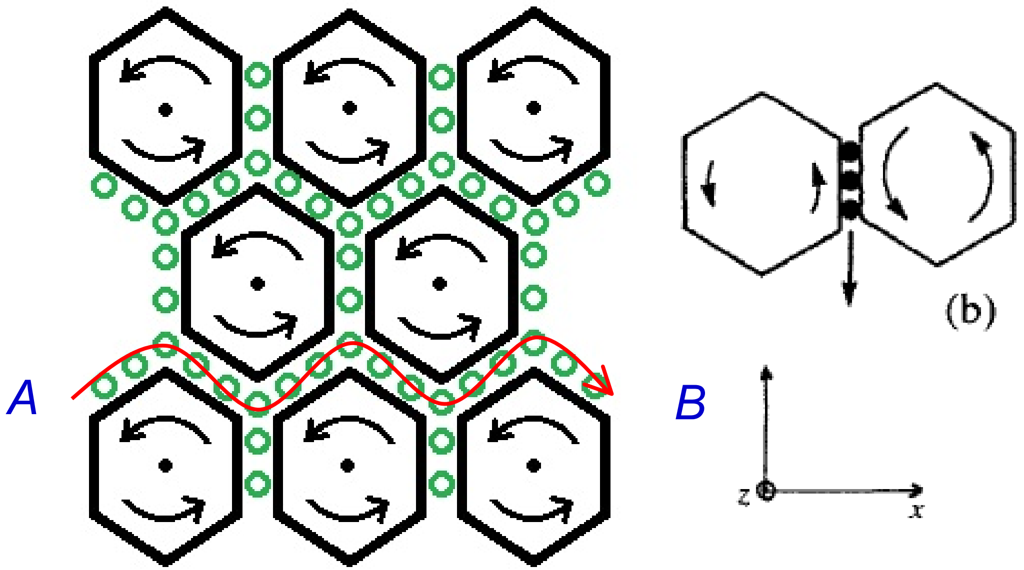
. . .



Particles between the vortices represent electric charges.

Kinematical Analysis

For example, if the vortices are parallel to the axis *Oz,* and if the rotation velocity H_z grows in the direction *Ox,* the shift occurs in the direction *Oy* at the rate $-\partial H_z/\partial x$. In general, the shift is given by $\nabla \times \mathbf{H}$, which is equal to the electric current density.



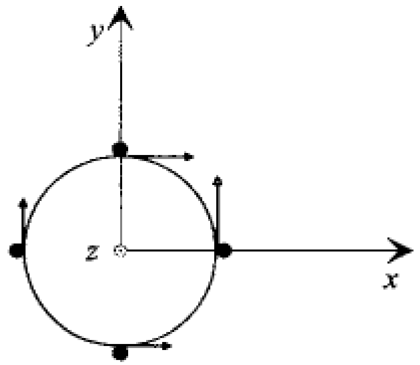
$$\nabla \times \mathbf{H} = \mathbf{j}$$

Maxwell therefore identified the stream of particles with the electric current.

Dynamical Analysis

As a result of the tangential action **T** of the idle wheels on the cells, there is a torque acting on each vortex. For example, the torque around Oz is proportional to $\partial_x T_y - \partial_y T_x$.

$$\frac{\partial}{\partial t}(\mu \mathbf{H}) = \nabla \times \mathbf{T}$$



Tangential action of four idle wheels on a vortex

According to the equality of action and reaction, the force **T** must be equal and opposite to the tangential action of the cell on the particles. Maxwell interpreted the latter action as the *electromotive force* $\boldsymbol{E_i}$ of magnetic origin acting on the current.

The condition that the work of the force $\boldsymbol{E_i}$ on the particles should be globally equal to the decrease of the kinetic energy of the cells determines the coefficient.

The curl of $\boldsymbol{E_i}$ is found to be equal to the time derivative of $-\mu \mathbf{H}$

$$\nabla \times \mathbf{E}_i = -\frac{\partial}{\partial t}(\mu \mathbf{H})$$

Statical Electricity

After Maxwell had interpreted "mechanically" Ampere's and Faraday's laws, he paused for about a year.

The model was not yet perfect, because it did not explain electrostatics. With no magnetic field, the electric field in a non conducting medium should create an electric polarization according to Faraday.

III. *On Physical Lines of Force.* *By* J. C. Maxwell, *F.R.S., Professor of Natural Philosophy in King's College, London**.

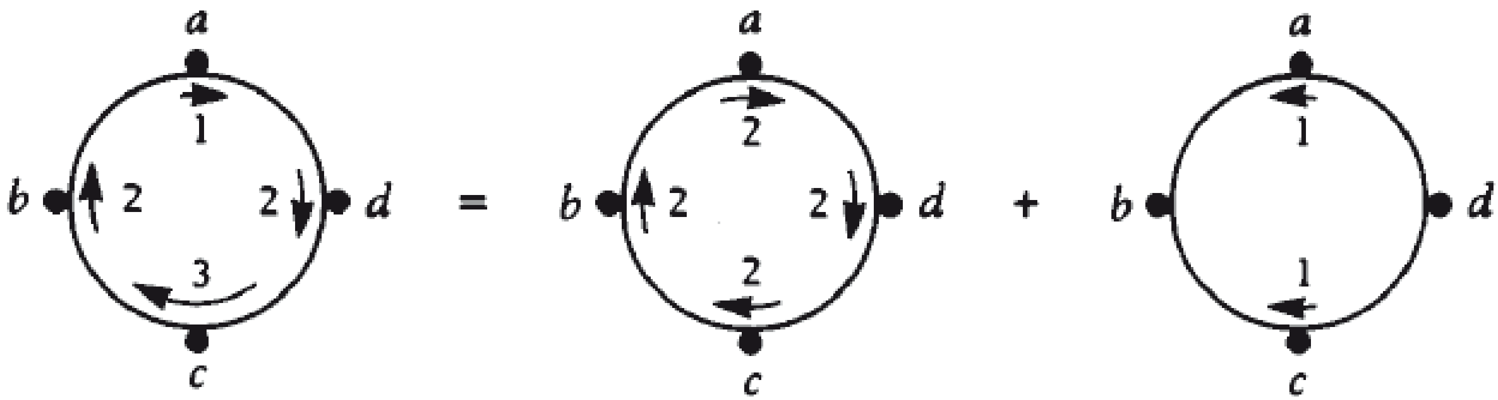
Part III.—*The Theory of Molecular Vortices applied to Statical Electricity.*

...

According to our theory, the particles which form the partitions between the cells constitute the matter of electricity. The motion of these particles constitutes an electric current; the tangential force with which the particles are pressed by the matter of the cells is electromotive force, and the pressure of the particles on each other corresponds to the tension or potential of the electricity.

...

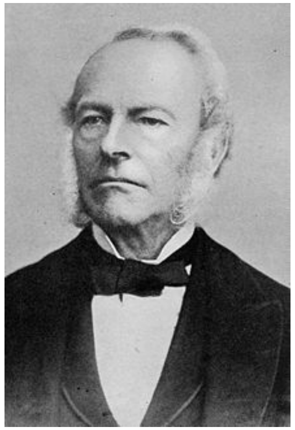
Stokes's Decomposition of a Displacement Field



Actual displacement

Rigid rotation

Pure deformation

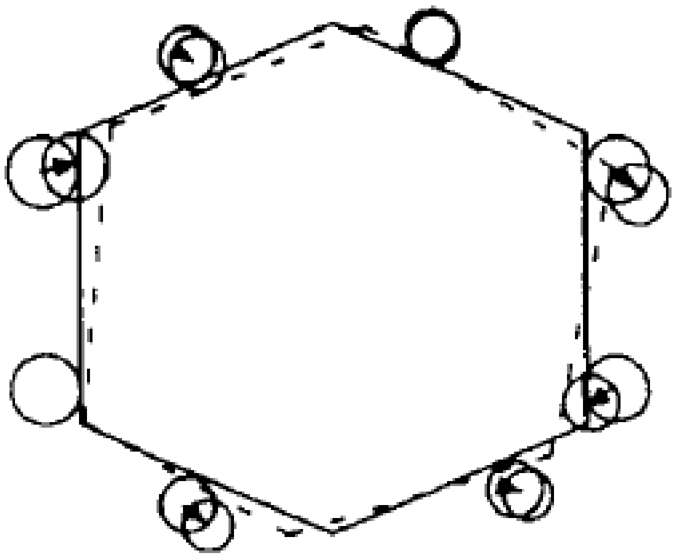


Sir George Gabriel Stokes
1st Baronet FRS
(England, 1819–1903)

By allowing the vortices to be deformable
Maxwell was able to explain electrostatic, too.

“On Physical Lines of Force” Part III.

Maxwell assumed that the vortices were elastic. When the electric field exerts a force to the fixed charge balls, they tend to distort the vortices and the charges are displaced from their stationary locations by a distance proportional to the electric force. In this way the electric force is transferred through the medium because of the elasticity.



The “**electric displacement**” is given by **D**=ε**E**, where the constant “ε” depends on the elasticity of the vortex fluid. Maxwell found that the energy density stored in the elastic displacement was of the form $\mathbf{E}\cdot\mathbf{D}/2$.
When he computed the energy corresponding to two point charges, he obtained the classical result given by Coulomb’s force.

“On Physical Lines of Force” Part III.

Because in Maxwell’s model the time derivative of the electric displacement is equivalent to an electric current, Maxwell rewrote Ampère’s equation as

$$\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$$

Even in an insulating medium one could have “currents” !!!

The structure now was elastic: it acted more like a “jelly” in which a disturbance would propagate as a transverse wave.

Maxwell showed that the propagation velocity of such waves was equal to the ratio of the electromagnetic and electrostatic units of the electric charge, which was already measured by Weber and Kohlrausch and was found to be equal to the value of the velocity of the light in air (known at that time).

Maxwell concluded that: “We can scarcely avoid the inference that light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.”

VIII. *A Dynamical Theory of the Electromagnetic Field.* By J. CLERK MAXWELL, *F.R.S.*

Received October 27,—Read December 8, 1864.

...

(2) The mechanical difficulties, however, which are involved in the assumption of particles acting at a distance with forces which depend on their velocities are such as to prevent me from considering this theory as an ultimate one, ...

...

(3) The theory I propose may therefore be called a theory of the *Electromagnetic Field*, because it has to do with the space in the neighbourhood of the electric or magnetic bodies, and it may be called a *Dynamical* Theory, because it assumes that in that space there is matter in motion, by which the observed electromagnetic phenomena are produced.

(4) The electromagnetic field is that part of space which contains and surrounds bodies in electric or magnetic conditions.

...

We have therefore some reason to believe, from the phenomena of light and heat, that there is an æthereal medium filling space and permeating bodies, capable of being set in motion and of transmitting that motion from one part to another, and of communicating that motion to gross matter so as to heat it and affect it in various ways.

...

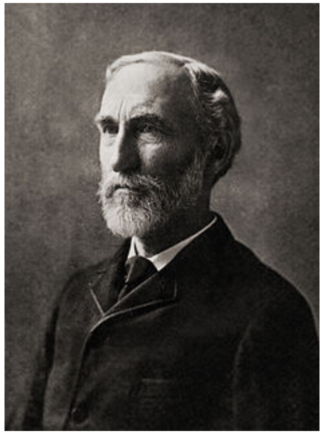
“A Dynamical Theory of the Electromagnetic Field”

Maxwell’s equations in modern Gibbsian vector notation

A	$C = K + \partial D / \partial t$	eq. of total currents
B	$\mu H = \nabla \times A$	eq. of magnetic force
C	$\nabla \times H = C$	eq. of currents
D	$E = v \times B - \partial A / \partial t - \nabla \Phi$	eq. of electromotive force
E	$E = kD$	eq. of electric elasticity
F	$E = \rho K$	eq. of electric resistance
G	$\nabla \cdot D = e$	eq. of free electricity
H	$\nabla \cdot K = -\partial e / \partial t$	eq. of continuity

(Bucci)

Maxwell also discusses the mechanical action in the field and gives mechanical force on a moveable conductor, a magnet, an electrified body.



Josiah Willard Gibbs
(United States, 1839 – 1903)

“Direct Action at Distance” Versus “Contact Action”

In Weber’s electrodynamics what matter are the charges, their distances, relative velocities and accelerations. The particles directly interact (the interaction is instantaneous).

In Maxwell’s picture of the physical world charges and magnets do not interact directly between them, but only through a medium, the omnipervasive “*luminiferous aether*”. Each charge would then interact only with the electromagnetic field produced by the *aether*.

“A treatise on Electricity and Magnetism” (1873)

In the preface Maxwell expresses his great admiration for Weber’s work.

…

Great progress has been made in electrical science, chiefly in Germany, by cultivators of the theory of action at a distance. The valuable electrical measurements of W. Weber are interpreted by him according to this theory, and the electromagnetic speculation which was originated by Gauss, and carried on by Weber, Riemann, J. and C. Neumann, Lorenz, &c. is founded on the theory of action at a distance, but depending either directly on the relative velocity of the particles, or on the gradual propagation of something,

…

J. C. Maxwell, *A treatise on Electricity and Magnetism*, Clarendon Press Series, 1873.

“A treatise on Electricity and Magnetism” (1873)

However, later Maxwell asserts that he has another vision of the physical word.

…

These physical hypotheses, however, are entirely alien from the way of looking at things which I adopt, and one object which I have in view is that some of those who wish to study electricity may, by reading this treatise, come to see that there is another way of treating the subject, which is no less fitted to explain the phenomena, and which, though in some parts it may appear less definite, corresponds, as I think, more faithfully with our actual knowledge, both in what it affirms and in what it leaves undecided.

…

J. C. Maxwell, A treatise on Electricity and Magnetism, Clarendon Press Series, 1873.

“A treatise on Electricity and Magnetism” (1873)

…

In a philosophical point of view, moreover, it is exceedingly important that two methods should be compared, both of which have succeeded in explaining the principal electromagnetic phenomena, and both of which have attempted to explain the propagation of light as an electromagnetic phenomenon, and have actually calculated its velocity, while at the same time the fundamental conceptions of what actually takes place, as well as most of the secondary conceptions of the quantities concerned, are radically different.

…

“The main motivation behind Maxwell’s effort was philosophical even metaphysical, *i.e.*, his adherence to a world view alternative to the dominant one, introduced by Michael Faraday in connection with his studies on electromagnetic induction and polarization.” (Bucci)

Nevertheless, Maxwell never knew that Weber’s electrodynamics failed to account for the mechanism of electromagnetic radiation in vacuum.

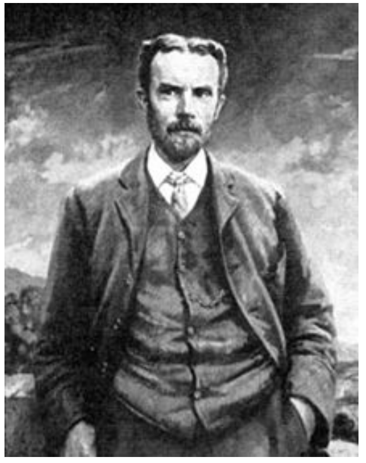
Retarted Electromagnetic Potentials

In 1867 **Lorenz** introduced the retarded electromagnetic potentials. By generalizing the Riemann’s intuition, Lorenz proposed that the magnetic vector potential were retarded, too.

Ludwig Lorenz
(Denmark, 1829-1891)

Maxwell's Equations in Vector Form

The Maxwell's equations in the nowadays vector notation were formulated by **Heaviside** in 1886.



Oliver Heaviside
(England, 1850-1925)

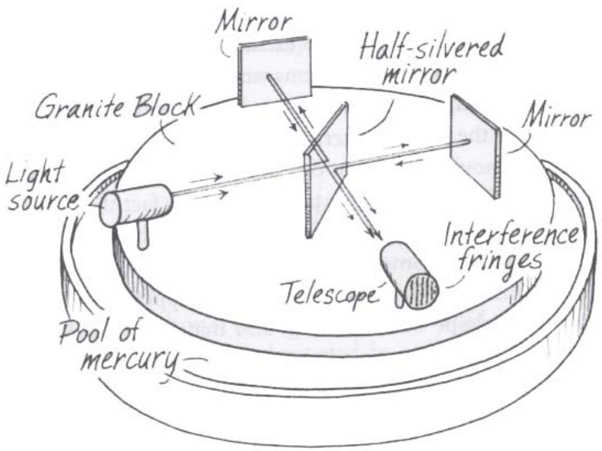
Michelson and Morley's Experiment

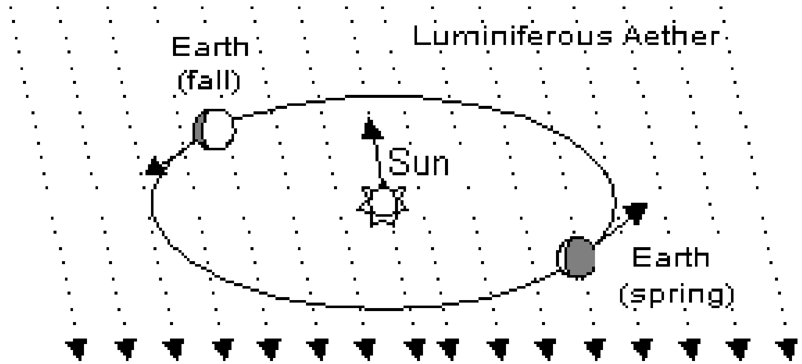
The result of the **Michelson** and **Morley's** experiment (1887) was a strong evidence against the "*luminiferous aether*".

Albert Abraham Michelson
(United States, 1852-1931)

Edward Williams Morley
(United Sytates, 1838-1923)

The Earth should move through a "medium" of *aether* that carries light.





Michelson and Morley tried to measure the "*aether* wind with a Michelson interferometer mounted on a granite block floating in a pool of mercury.

It was a failure !

Hertz's Theory

In 1884 **Hertz**, Helmholtz's pupil, published a paper in which starting from the older action-at-distance theory (Weber's electrodynamics) proceeded to obtain Maxwell's equations in an alternate way.

Heinrich Rudolf Hertz
(Germany, 1857-1894)

Hermann Ludwig Ferdinand von Helmholtz
(Germany, 1857-1894)

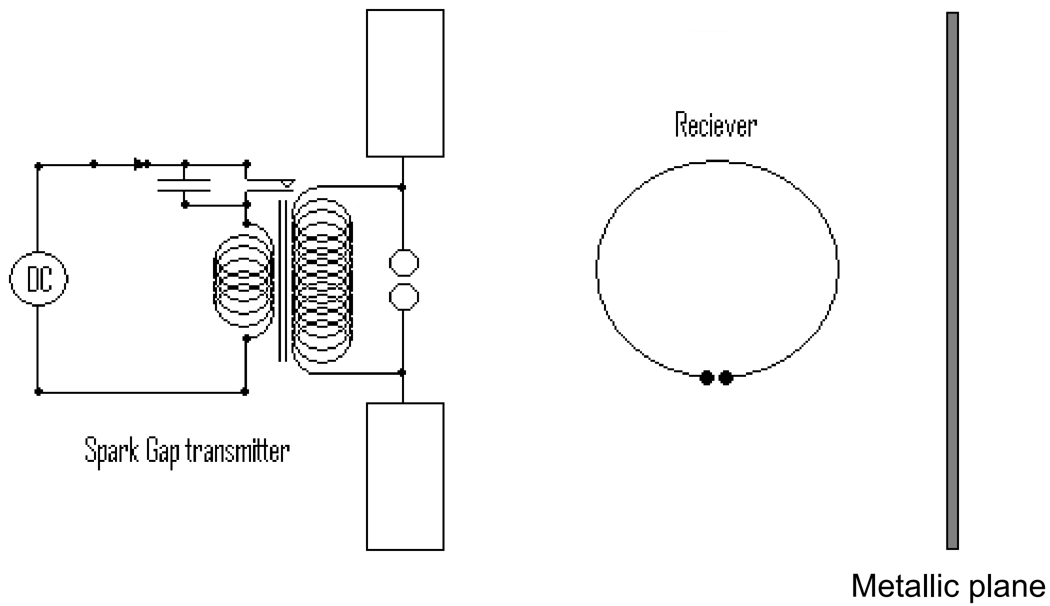
Hertz avoided both the mechanical models of Maxwell and his formulation of displacement current by using successive approximation from the earlier action-at-distance theory together with a principle of the "unity of the electric force".

However, Hertz's conclusions were: "If the only choice lay between the usual system and Maxwell's, than the latter has undoubtedly the advantage".

This results has probably prompted Hertz to search vigorously for the experimental evidence of Maxwell's electromagnetic theory.

Hertz's Experiment on Radio Waves

In 1888 **Hertz** confirmed experimentally the existence of electromagnetic waves, other than light (at radiofrequency) and so validated the Maxwell's Equations.

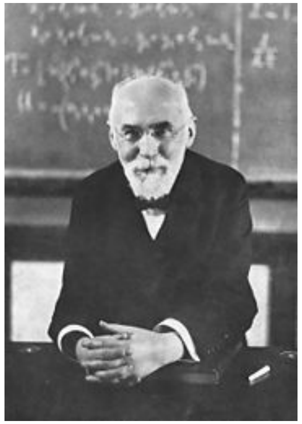


In contrast to the 1884 paper, in his 1890 paper "*On the fundamental Equations of Electromagnetics for bodies at Rest*" Hertz postulated Maxwell's equations, rather than deriving them in an alternative form.

Lorentz's Force Law

In 1895 **Lorentz** introduced his force law.

The Maxwell's equations do not furnish the force exerted on the charges and currents, they only give the electromagnetic field generated by them.



Hendrik Antoon Lorentz
(Honland, 1853-1928)

The **Lorentz's force** combined to **Newton's equation** tell us how the charges react in presence of an electromagnetic field.

Electron Discovery

In 1897 **Thomson** discovered the electron.

Sir Joseph John "J. J." Thomson
(England, 1856-1940)

Liénard and Wiechert's Retarded Potentials

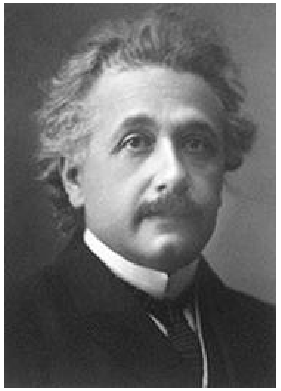
In 1898 **Liénard** published a paper on retarded electromagnetic potentials due to corpuscular charges. In 1900 **Wiechert** published a paper on the same subject.

Alfred-Marie Liénard
(France, 1869-1958)

Emil Wiechert
(German, 1861-1928)

Theory of Special Relativity

The epochal paper of **Einstein** on the theory of special relativity (1905) causes the definitive abandonment of the ether theory.



Albert Einstein
(German,1879-1955)

Special Relativity showed that Maxwell's equations do not require an *aether*.

In the meantime the atomic theory of matter has been developed and the quantum revolution has started.

Success of Maxwell's Equations

Nowadays we do not speak of ***aether*** anymore, but **electromagnetic field** has taken its place. So each charge generates an electromagnetic field, this electromagnetic field propagates at a finite speed from the charge and interacts with the other charges when it reaches them.

The electromagnetic field is governed by **Maxwell's Equations**, which stood unchanged and are still those created by his genius 150 years ago.

Limits of Maxwell's Equations

Description of the action of a point charge upon itself and consequently an infinity in the energy of the electromagnetic field.

The classical electromagnetism has been developed at the time when it was considered appropriate to treat electric charges as a continuous substance.

"It is not obvious that general acceptance in the early 1800's of the principle of the atomicity of electric charge would have led to the field concept in its present form." (Wheeler, Feynman)

J. A, Wheeler, R. P. Feynman, Classical Electrodynamics in Terms of Direct Interparticle Action, Reviews of Modern Physics, 21, pp. 425-433, 1949.

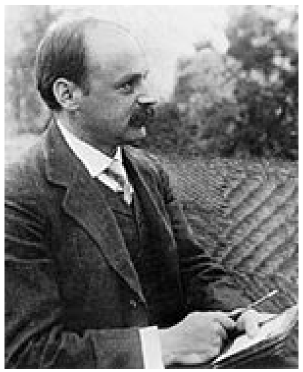
Direct Interparticle Action

Wheeler and **Feynman** (1949) showed that Schwarzschild's direct interparticle action is equivalent to a *modified field theory* in which:

♦ The motion of a given particle is determined by the sum of fields produced by - or adjunct to - every particle other than the given particle.

♦ The field adjunct to a given particle is uniquely determined by the motion of that particle, and is given by half the retarded plus half the advanced solution of the field equations of Maxwell for the point charge in question.

John Archibald Wheeler
(United States, 1911-2008)



Karl Schwarzschild
(German, 1873 – 1916)

Richard Phillips Feynman
(United States, 1918-1988)

J. A, Wheeler, R. P. Feynman, Classical Electrodynamics in Terms of Direct Interparticle Action, Reviews of Modern Physics, 21, pp. 425-433, 1949.

Vacuum Fluctuations

Quantum electrodynamics prevent simultaneous vanishing of electric and magnetic energies.

The ground state of the electromagnetic field in *vacuum* has a nonzero absolute energy with *fluctuating* electromagnetic field amplitude.

Are vacuum fluctuations an *aether*?

Thank you for the attention